VIBRATION RESPONSE OF A MULTISUPPORTED BEAM IN PARALLEL FLOW

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By

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CERTIFICATE

This is to certify that the work entitled 'Vibration Response of a Multisupported Beam in Parallel flow' by Sankar Dhar has been carried out under our supervision and has not been submitted else where for the award of a degree.

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$C(\tau)$	Auto Correlation Coefficient
C_P	Convection Velocity
CV	Non-dimensional Convection Velocity = $(C_P \ell \sqrt{\frac{m}{EI}})$
E	Young's Modulus
f	Frequency in Hz
$H(f)$	Frequency Response Function
I	Moment of Inertia
K_r	Rotational Stiffness at each support of the beam
k_r	Non-dimensional stiffness parameter [= $(K_r \ell/EI)$
K	Wave Number of Loading
ℓ	Span Length
m	Beam Mass Per Unit Length
$P(f)$	Power Spectral Density Function
$R_{pp}(\tau)$	Auto Correlation Function of Pressure
$R_{p_1p_2}(\tau)$	Cross Correlation Function of Pressure
$R_{xx}(\tau)$	Auto Correlation Function of Vibration Response
$S_{PP}(\Omega)$	Power Spectrum of Pressure
$S_{xx}(\Omega)$	Power Spectrum of Vibration
β_{AB}	Rotation at station 'A' due to unit moment at station 'B'

μ_r	Attenuation Constant
μ_i	Phase Constant
μ	Propagation Constant $(\mu_r + i\mu_i)$
ω	Circular Frequency in Radian
Ω	Frequency Parameter = $(\omega \ell^2 \sqrt{\frac{m}{EI}})$
η	Damping loss factor.

SYNOPSIS

Of the

Disertation on

VIBRATION RESPONSE OF A MULTISUPPORTED BEAM IN PARALLEL FLOW

Submitted in Partial Fulfilment of the requirement for the Degree of MASTER OF TECHNOLOGY IN MECHANICAL ENGINEERING

by

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Vibration of a rod, supported at regular interval, subjected to a parallel flow is investigated experimentally. Statistical measurements are carried out for both the exciting pressure and the vibration response. The exciting pressure field is modelled as a random pressure wave convected at a particular velocity. The convection velocity is also obtained experimentally and is found to be nearly equal to the flow velocity. The system response function for such loading, obtained from the experimental data, is compared with theoretical predictions. The theory is based on a wave approach suitable for periodic structures. Good correlation is obtained if the pressure field is measured on the specimen.

CHAPTER - I

INTRODUCTION

1.1 Introduction:

Flow induced vibration is a wide field associated with engineering structure placed in flowing fluid. Energy needed for this type of vibration is derived from flowing fluid around it. The vibration may be caused by far field noise or near field noise. In the first case turbulence, eddies and cavitation etc. play major part while in the second case it is mainly due to pressure fluctuation in boundary layer of the submerged body.

Flow induced vibration problems were studied earlier for fatigue damage of nuclear reactor core where the flow velocity is comparitively high for higher rate of heat transfer. Flow may be parallel or perpendicular to the axis of the rod. For cross flow which is mainly important in heat exchanger, the vibration is due to the far field noise. The problem has been widely studied by several workers and empirical relations are available as design guide. However, in the case of parallel flow some theoretical models have been attempted using statistical methods. Due to lack of information about the flow field around the rod, it is assumed that the rod is excited

by a random time varying pressure field. Statistical correlations have been derived to obtain a qualitative idea in this field.

The present work is attempted to study the vibration of long nuclear fueld rods (PWR reactor) which are supported periodically by grid plates. Effect of continuous errosion of baffles resulting in enlargement of baffle holes and notching of fuel rod due to hammering and sawing of the vibrating rods are observed. A study of the problem requires the knowledge of the response of a periodic structure under random convected pressure loading.

1.2 Review of Previous Works:

The interest in the area of flow induced vibration was reactivated by Ashley and Haviland [1] in connection with Trans Arabian pipe lines. A lot of theoretical work in this area has been taken up since then. A nice comprehensive review of the literature is given by Paidousis and Issid [2]. A brief review of the experimental work is given below. Burgreen et al. [3] studied the vibration of rods with different end conditions in parallel flow and it was observed that the vibration is of self excited type. The rods vibrate at their own natural frequencies independent of flow velocity. Quinn [4] also measured the vibration of a submerged rod in water at

temperature ranging from 21 to 61°C. The experiments were also carried out for two phase flow. Paidousis [5] performed some experiments on a single rod and a bunddle of rods in parallel flow. Based on his experiment he revised the empirical relation previously derived by Burgreen et al. Reavis [6] in 1969 offered an attractive approach to the problem postulating that the vibration is excited mainly by boundary layer noise. He compared his results with the theoretical solution given by Thomson [7] for the first mode response of a pinned beam to a distributed random loading. The forcing function as given by Bakewell [8] was used in the previous work. Bakewell suggested that the pressure fluctuation is convected down stream and is a function of frequency. Ram Jiyavan [9] tried to derived some statistical correlations of the pressure characteristics of parallel flow over a thin rod. He also observed the spectral variation of pressure field with flow velocities. All these works that have been done in the field of parallel flow induced vibration are on single span rod or tube. But when the nuclear fuel element is supported intermediately the problem is modelled as a periodic structure under random distributed loading.

The most common method for analysing a structure subjected to random loading is the modal method. In this method, it is assumed that the modes are independent of each other which is not true for a periodic structure. Moreover, so many modes

are associated with periodic structure with many supports that the method becomes unwieldy. Lin [10] applied transfer matrix method to solve the problem of beam with many supports. The use of finite element method for solving such problems has also been attempted [11]. Recent application of wave propagation theory in periodic structure by Mead [12] made the problem easier for analysis. The main advantage of this method is its simple analysis and the negligible computational time which is independent of number of supports. Sen Gupta [13] developed a graphical method, using wave approach to determine the natural frequencies of a periodic beam. Mead [14] used the wave approach to determine the response of a periodic structure under convected harmonic loading. He showed that the 'coincidence' occurs at a frequency when the convection velocity of the pressure is equal to the phase velocity of the free waves propagating the structure. The limitation of the theory is that a closed form solution is possible only when the beam is uniform. However, Mead and Mallik [15] have developed an approximate method using 'assumed mode' technique to determine the response of an infinite periodic structure with non-uniform element. Rao [16] applied this method to find the response of a finite periodic beam.

1.3 Scope of Present Work:

The present work deals with the vibration response of a

long rod which is supported at regular intervals. Auto correlation functions and power spectra of both pressure (excitation) and vibration (response) have been obtained. With these results frequency response functions of the system have been computed with different flow velocities. Results have been obtained in various spans of the rod. The experimentally obtained results are compared with the theoretical predictions obtained by using wave approach. Auto correlation coefficient of both pressure and response have been computed using Honeywell SAI-48 correlator. Power spectral density functions have been computed from auto correlation results on DEC-10 using the well known relation.

$$P(f) = \lim_{\tau_m \to \infty} \int_{-\tau_m}^{\tau_m} c(\tau)\, e^{-i2\pi f\tau}\, d\tau \qquad (1.1)$$

where $c(\tau)$ is the auto correlation coefficient and f denotes the frequency in Hz.

With N discrete values of $c(\tau)$ spaced at Δt apart, the power spectral density function can be expressed as

$$P(f) = \frac{1}{N} c(o) + \frac{2}{N} \sum_{j=1}^{M} c(j\Delta t) \cos 2\pi f j \Delta t$$

$$= \Delta t\, c(o) + 2\Delta t \sum_{j=1}^{M} c(j\Delta t) \cos 2\pi f j\Delta t \qquad (1.2)$$

Having power spectra of both excitation (pressure) and response (vibration), the system frequency response function can be computed with the relation

$$S_{xx}(f) = |H(f)|^2 S_{PP}(f) \qquad (1.3)$$

where $S_{xx}(f)$ = power spectrum of response.

$S_{PP}(f)$ = power spectrum of excitation.

CHAPTER - II

EXPERIMENTAL SET UP

2.1 Introduction:

A schematic view of the experimental set up is shown in Fig. (2.1). The horizontal test loop consists of several removable sections of 15.5 cm dia C.I. pipe. A centrifugal pump (22 H.P.) delivers water to the test loop. Flow of the water can be controlled by three valves (V_1, V_2, V_3). The outlet water is recirculated to the reservoir again.

The test loop has been designed for minimum far field noise, The loop consists of two flow straightners, one AFT filter and two sets of acoustical filters (F_1, F_2). The acoustical filters are placed in upstream and down stream of the test section to take care of up stream noise and reflected noise as well. The outlet water passes over a V notch while flowing through the recirculating channel. Detailed discussion of the test loop is given in ref. [9].

2.2 Test Section:

The test section consists of a 2m long, 15.5 cm. I.D. M.S. pipe placed between filters F_1 and F_2. It contains a 5 ft. long test element periodically supported from the outer pipe by pointed supports. The supports are designed to simulate the simple support condition as nearly as possible. The test

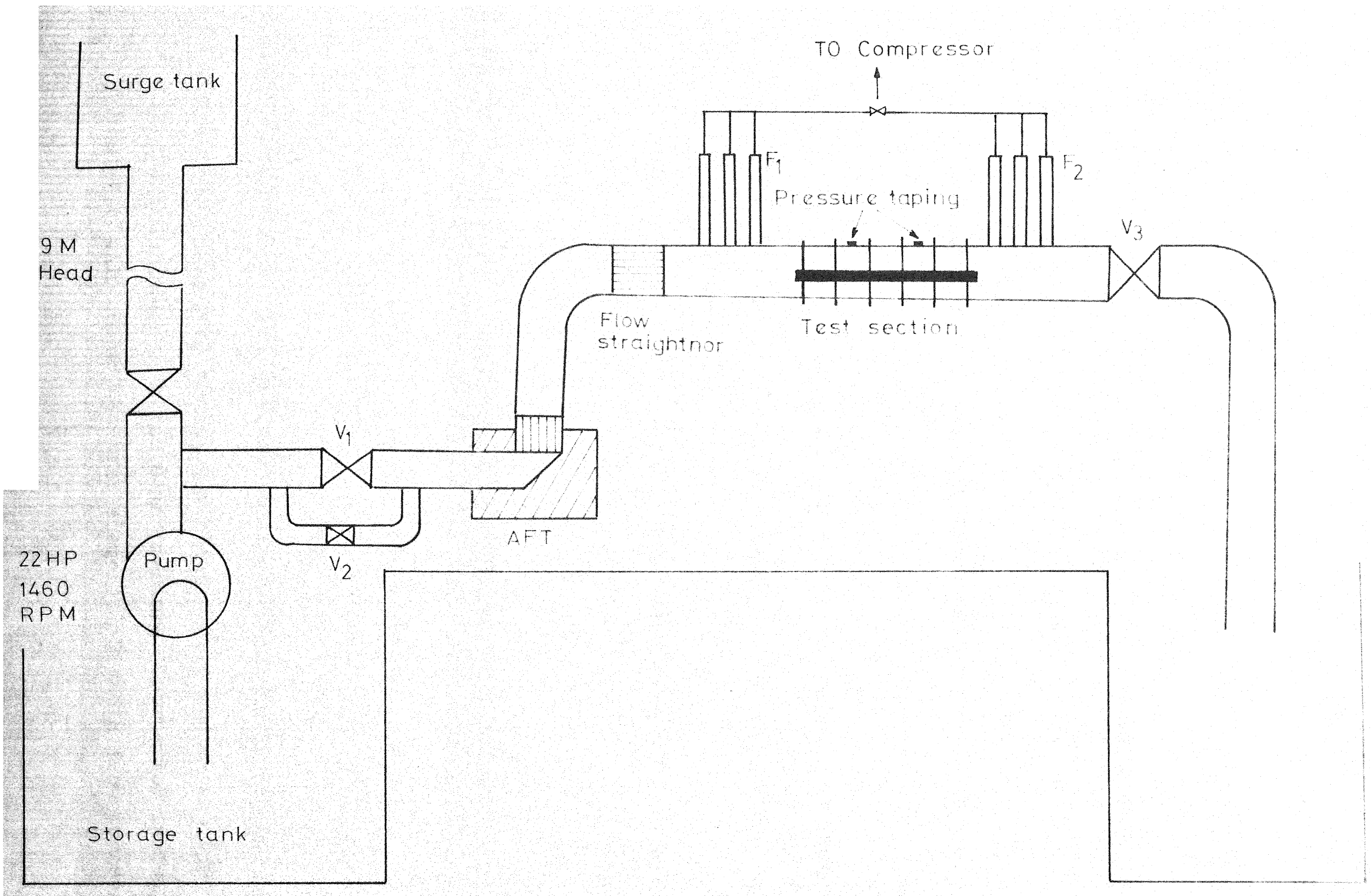


FIG 2.1 SCHEMATIC DIAGRAM OF TEST LOOP

element is push fitted with wedge shaped rings at 1 ft. interval to provide a line support on the test element. Each ring is supported by four pointed end bolts so that the test element rests coaxially with outer pipe. The test element is made of five parts of equal length. Each part is connected to its adjacent part by means of threaded socket. The support locations are so chosen that the sockets are at middle of each bay.

The sockets are instrumented with strain gauges to measure the vibration response at the centre of each bay. The above arrangement has been done for some specific advantages.

(1) It is easy to instrument the small socket rather than the long test element.

(2) One instrumented socket can be changed from one bay to another so that we need not instrument all the bays at a time.

2.3 Instrumentation and Signal Conditioning:

The experimental procedure consists of two parts:

(a) Measurement of pressure which is the excitation function.

(b) Measurement of vibration response.

For measurement of pressure, two pressure transducers (ENDEVCO model 2520) have been mounted on the walls of the outer pipe with the assumption that the same pressure fluctuation is felt on the pipe wall as on the test element. Signals from pressure

transducers are fed to the charge amplifiers. From charge amplifier the signal is taken to the correlator via a low pass filter.

Vibration measurement has been done with strain gauges. Since the amplitude of vibration is very low semi-conductor strain gauges are used due to their high sensitivity and gauge factor. Two strain gauges have been mounted on the top and bottom of each socket. Leads of the strain gauges have been taken out through the hollow test element so that the flow over the test element is not disturbed. Leads of the strain gauges are taken to a Budd strain indicator (Model P-350) to balance a half bridge circuit. The output from the strain indicator is fed to a linear amplifier in order to amplify the signal. This amplified signal is fed to the correlator via a low pass filter.

2.4 Experimental Procedure:

2.41 Sampling:

Since SAI-48 correlator is a digital unit, choosing proper sampling time is important. Two types of difficulties may arise for improper choosing of sampling time. If the sampling time is too short, to obtain the necessary information from the data will involve too much time and increase the cost of calculation. On the other hand sampling at points which are too far apart will lead to statistically poor data

especially for low and high frequency components of the original data. This is called aliasing. In general sampling time is chosen from the knowledge of highest frequency present in original record. The relation between highest frequency and sampling time is given by $f_c = \frac{1}{2h}$, where f_c is called Nyquist folding frequency and h is sample time interval. Frequency above Nyquist frequency will be folded back. In our experiment all the data have been taken below 100 cps. Therefore the proper sampling time to avoid aliasing is 5 ms. To be on the safe side so that no frequency component is lost the sampling time in most of the recording is taken to be 1 ms. SAI-48 Correlator has a series of sampling time ranging from 1 μs to 2 sec and gives an output of 400 correlated data points.

2.42 Measurement of Correlation functions:

As mentioned earlier that the experimental procedure consists of two parts: (1) Measurement of pressure (2) Measurement of vibration response. For each part auto correlation functions have been computed in SAI-48 correlator. SAI-48 performs correlation for 400 points. Signals from transducers are fed to either channel of the correlator. Appropriate delay is introduced in one of the channels for computing the correlation function. Digital output from correlator are taken to a digital computer (DEC-10) for computing power spectral density function. For measurement of convection velocity cross correlation functions have been computed between two pressure trans-

ducers placed axially apart. Cross correlation function accurately determines the delay time (T_d) of the signal between two pressure transducers. Convection velocity can be found from relation $C_p = \frac{d}{T_d}$, where C_p denotes convection velocity and d represents transducers separation distance. The cut off frequency in most of the measurement is kept below 100 Hz because the strain gauges are unable to respond at higher frequencies. All these experiments have been done for different flow velocities. The flow velocity has been measured conventionally from the head of the water over a V notch while it was passing through the recirculating channel.

CHAPTER - III

THEORETICAL ANALYSIS

3.1 Introduction:

A periodic structure consists of identical elements connected in identical manner. A typical example of this structure is a multi supported continuous beam with identical supports at regular interval. The response of such structure was analysed in the past by modal method. When the beam contains many supports, so many modes are to be included that the modal analysis becomes cumbersome. Under such circumstances it is convenient to consider the structure as infinite so that discrete modes can be ignored. The motion can be conveniently analysed by a so called wave approach.

3.2 Fundamentals of Wave Approach:

A simple wave propagating along an one dimensional damped structure can be characterised by the wave number $K = 2\pi/\lambda$, which is nothing but the phase difference between wave motion at two points which are unit distance apart. The wave decays as it propagates. The amplitude ratio of two points which are unit distance apart is e^{δ}. The quantity $(\delta + iK)$ is called the propagation constant which defines the amplitude and the phase relation per unit length of the wave motion. Suppose an infinite beam with equispaced supports is excited at a

single point P by a harmonic force $P_o e^{i\omega t}$. The wave motion will propagate outwards from P. The displacements in any two bays may not be identical. However, the amplitude ratio and phase difference of two identical points in adjacent bays are same. μ_i represents the phase difference between two identical points in adjacent bays over a distance ℓ while e^{μ_r} represents the amplitude ratio.

The characteristic propagation constant is now $\pm(\mu_r + i\mu_i)$. The negative sign implies that the wave is decaying as it is propagating from left to right while the positive sign implies the reverse direction.

3.3 Properties of Propagation Constant:

Consider two adjacent bays A-B and C-D resting on rigid simple supports, restrained at each end by a rotational spring $K_r/2$ (Figure 3.1b). The total rotational stiffness at each support is K_r. Suppose the beam is subjected to free vibration. The moments at (r-1), r, (r+1)th supports are M_{r-1}, M_r, M_{r+1} respectively. These are all harmonically varying complex quantities. Let us consider the span A-B, slope of the beam at point B in span A-B may be expressed as

$$\theta_B = \beta_{BA} M_{r-1} + \beta_{BB} M_r \qquad (3.1a)$$

where β is called receptance function which is defined as follows

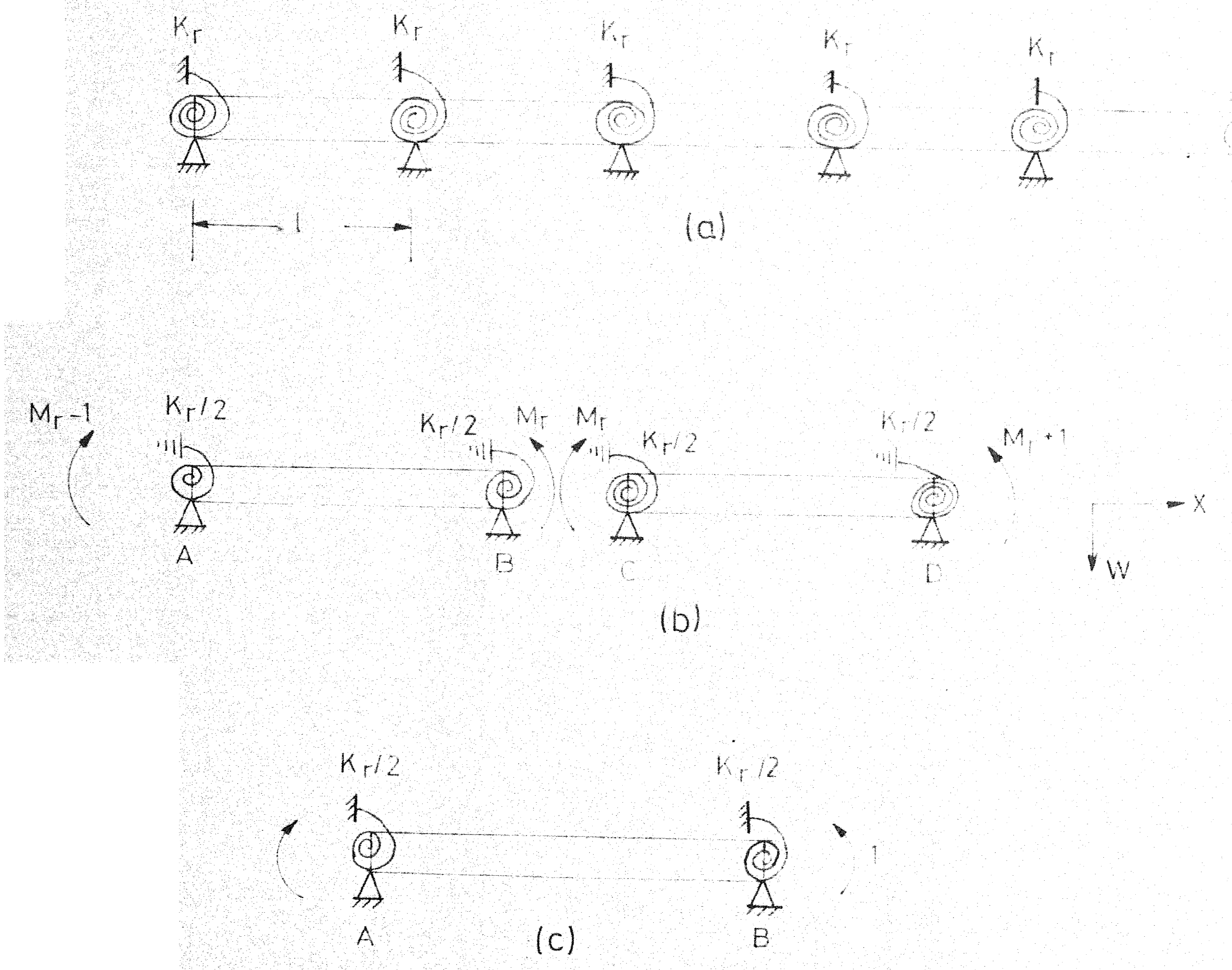


FIG. 3·1 PERIODIC BEAM ELEMENT

β_{BA} = slope at point B due to unit moment at point A.

β_{BB} = slope at point B due to unit moment at point B.

Similarly, if we consider the span C-D, the slope at point C

$$\theta_C = \beta_{CC} M_r + \beta_{CD} M_{r+1} \tag{3.1b}$$

If the beam is uniform and resting on identical supports then, $\beta_{BB} = -\beta_{CC}$ and $\beta_{BA} = -\beta_{CD}$.

For continuity of slope $\theta_B = \theta_C$. Hence, equating equations (3.1a) and (3.1b) and using above receptance relations, we get

$$M_{r-1} + \frac{\beta_{BB}}{\beta_{BA}} M_r + M_{r+1} = 0 \tag{3.2}$$

The above relation applies between two adjacent free bays. Such set of equations satisfy recurrence relations like

$$\left.\begin{aligned} M_{r+1} &= e^{\mu} \cdot M_r \\ M_r &= e^{\mu} \cdot M_{r-1} \end{aligned}\right\} \tag{3.3}$$

Substituting these relations in equation (3.2), we get

$$e^{\mu} + e^{-\mu} = -\frac{2\beta_{BB}}{\beta_{BA}}$$

or

$$\cosh\mu = -\frac{\beta_{BB}}{\beta_{BA}} \tag{3.4}$$

The right hand side of the equation is highly frequency dependent and complex in nature if damping is present in the

beam. For an undamped beam, if $-\frac{\beta_{BB}}{\beta_{BA}}$ is greater than +1, μ is entirely real. Therefore, the wave is decaying in nature but there is no phase difference between two adjacent bays. The wave is non propagating type. If $-\frac{\beta_{BB}}{\beta_{BA}}$ is less than -1, μ is of the form $\mu = \mu_r + i\pi$. The waves in two adjacent bays are in counter phase but still decaying in nature. If $-\frac{\beta_{BB}}{\beta_{BA}}$ is between -1 to +1 then μ is entirely imaginary. This means there is no decay in wave amplitude and wave can propagate. The frequency range in which $-\frac{\beta_{BB}}{\beta_{BA}}$ lies between -1 and +1 is called the propagation band.

3.4 Response of An Infinite Periodic Beam under Harmonic Convected Loading:

Consider an Euler-Bernoulli beam resting on periodic supports at distance ℓ apart and excited by a harmonic pressure field such that the force per unit length of the beam is

$$P(x,t) = P_o e^{i(\omega t - Kx)}$$

where K is the wave number and is equal to $\frac{\omega}{C_P}$ with C_P as the convection velocity. Phase difference between two points at a distance x apart is $-Kx$.

The motion of the beam is governed by the following equation

$$EI \frac{\partial^4 W}{\partial x^4} + m \frac{\partial^2 W}{\partial t^2} = P_o e^{i(\omega t - Kx)} \qquad (3.5)$$

If damping is present in the beam, then EI should be replaced by $EI(1+i\eta)$ where η = loss factor.

EI = flexural rigidity of the beam and m = mass per unit length of the beam.

Let us put $x = \xi\ell/2$, where ξ is the non-dimensional x co-ordinate, with the origin at the middle of the bay.

Assuming the steady state solution $W = \bar{W}(\xi)e^{i\omega t}$ and subtituting this in the governing equation one gets

$$16(EI/\ell^4)(1+i\eta)\,\frac{d^4\bar{W}}{d\xi^4} - m\,\omega^2\,\bar{W} = P_o e^{-iK\ell\xi/2} \qquad (3.6)$$

or

$$\frac{d^4\bar{W}}{d\xi^4} - \frac{m\,\omega^2\,\ell^4\,\bar{W}}{16\,EI(1+i\eta)} = \frac{P_o\,\ell^4}{16\,EI(1+i\eta)}\,e^{-iK\ell\xi/2} \qquad (3.7)$$

Let us introduce the following non-dimensional quantities

(a) frequency parameter $\Omega = \omega\ell^2\sqrt{\frac{m}{EI}}$

(b) loading parameter $P = \frac{P_o\ell^3}{EI}$

(c) convection velocity parameter $CV = C_P\ell\sqrt{\frac{m}{EI}}$

Substituting these quantities in equation (3.7), we get

$$\frac{d^4\bar{W}}{d\xi^4} - \frac{\Omega^2\,\bar{W}}{16(1+i\eta)} = \frac{P\,\ell}{16(1+i\eta)}\,e^{-i\frac{\Omega}{2CV}\xi} \qquad (3.8)$$

The solution of the equation is

$$\bar{W} = \sum_{n=1}^{4} A_n\,e^{\lambda n\,\xi} + P.I \qquad (3.9)$$

$$P.I = \frac{1}{\frac{\Omega^4(1+i\eta)}{CV^4} - \Omega^2} \, e^{-i\frac{\Omega}{2CV}\xi}$$ where P_ℓ is taken to be 1.

λ_n are the four fourth roots of the quantity $\frac{\Omega^2}{16(1+i\eta)}$.
A_n's are the constants to be found out from the boundary conditions. Now consider the motion of a free bay. Since there is no loading in free bay the right hand side of the equation 3.8 is absent. The solution in such case is

$$\bar{W} = \sum_{n=1}^{4} A_n e^{\lambda_n \xi} \qquad (3.10)$$

To find the receptance functions β's, apply a unit non dimensional harmonic moment at end A of the span AB figure (3.1c). The boundary conditions are:

$$\begin{aligned} \bar{W}(-1) &= 0 \\ \bar{W}(+1) &= 0 \\ -\bar{W}''(-1) &= \frac{1}{(1+i\eta)} - \frac{1}{2}\frac{k_r}{(1+i\eta)}\bar{W}'(-1) \\ -\bar{W}''(+1) &= \frac{1}{2}\frac{k_r}{(1+i\eta)}\bar{W}'(+1) \end{aligned} \qquad (3.11)$$

Here we have considered that the supports are transversely rigid. The quantity k_r is non dimensional rotational stiffness at each support

$$k_r = \frac{K_r \ell}{EI}$$

Applying above boundary conditions A_n can be solved uniquely. Now the receptance β_{AA} and β_{BA} are given by

$$\beta_{AA} = \overline{W}'(-1) = \sum_{n=1}^{4} A_n \lambda_n e^{-\lambda_n} = -\beta_{BB} \qquad ,$$

$$\beta_{BA} = \overline{W}'(+1) = \sum_{n=1}^{4} A_n \lambda_n e^{\lambda_n} = -\beta_{AB} \qquad (3.12)$$

In general, λ_n's are complex and frequency dependent. Therefore, receptance functions are also complex and frequency dependent. Using these values of receptance function propagation constant (μ) can be calculated from equation (3.4).

3.5 Forced Wave Response in An Infinite Beam:

In order to find the forced wave response generated in an infinite beam, let us consider the equation (3.9) where P.I. is not zero together with the boundary conditions given below.

$$\overline{W}(-1) = 0$$

$$\overline{W}(+1) = 0 \qquad (3.13)$$

Slopes at either end should satisfy $\overline{W}'(+1) = e^{-iK\ell}\,\overline{W}'(-1)$ and bending moment relation is given by

$$EI\,(1+i\eta)\,\overline{W}''(+1) + \frac{1}{2} k_r \overline{W}'(+1) = e^{-iK\ell}\,EI(1+i\eta)\overline{W}''(-1)$$

Using above boundary condition A_n's can be determined and the displacement is given by equation (3.9).

3.6 Response of A Finite Periodic Beam to Convected Pressure Field:

The response of a finite periodic beam consists of two parts:

(a) Forced wave motion generated in an infinite periodic beam.

(b) Response corresponding to reflection of waves from extreme ends which are essentially free waves.

The solution of part (a) is given in the previous section. The solution of part (b) for free waves can be written as

$$W_f(\xi) = \sum_{n=1}^{4} B_n e^{\lambda_n \xi}$$

B_n's are the constants to be solved from the following boundary conditions.

$$W_f(-1) = 0$$

$$W_f(+1) = 0$$

Slopes at either end is related by the relation

$$W'(+1) = e^{\mu} W'(-1)$$

The above three boundary conditions can not determine four constants uniquely. However, if we consider $\frac{B_1}{B_1} = b_1 = 1, \frac{B_2}{B_1} = b_2, \frac{B_3}{B_1} = b_3$ and $\frac{B_4}{B_1} = b_4$, then B_n's are determined uniquely apart from a common factor which comes in the amplitude of the wave.

The above boundary conditions can be rewritten in the form

$$\sum_{n=2}^{4} b_n e^{-\lambda_n} = -e^{-\lambda_1}$$

$$\sum_{n=2}^{4} b_n e^{\lambda_n} = -e^{\lambda_1} \tag{3.14}$$

$$\sum_{n=2}^{4} b_n \lambda_n (e^{\lambda_n} - e^{\mu} e^{-\lambda_n}) = -\lambda_1 (e^{\lambda_1} - e^{\mu}.e^{-\lambda_1})$$

From these equations with given μ_+ and μ_- for positive and negative going waves two sets of values b_{n+} and b_{n-} can be found.

Total response of a beam is given by

$$W_T(\xi) = \bar{W}(\xi) + W_{f+}(\xi) + W_{f-}(\xi)$$

Where $\bar{W}(\xi)$ is the forced response in an infinite beam

$$W_T(\xi) = \bar{W}(\xi) + B_1 \sum b_{n+} e^{\lambda_n \xi} + B_2 \sum b_{n-} e^{\lambda_n \xi} \tag{3.15}$$

The unknown constant B_1 and B_2 can be calculated from extreme boundary conditions. For a five span beam, these are as follows

$$EI(1+i\eta)\, W_T''(-1) = K_r \ell\, W_T'(-1)$$

$$-EI(1+i\eta)\, W_T''(+1)_5 = K_r \ell\, W_T'(+1)_5 \tag{3.16}$$

Substituting the value of W_T from equation (3.15), we get

$$EI(1+i\eta)\ [W''(-1)+W''_{f_+}(-1)+W''_{f_-}(-1)] = K_r\ell\ [W'(-1)+W'_{f_+}(-1)+W'_{f_-}(-1)]$$

$$-EI(1+i\eta)[e^{-5iK\ell}\ W''(-1) + e^{5\mu_+}\ W''_{f_+}(-1) + e^{5\mu_-}\ W''_{f_-}(-1)]$$

$$= K_r\ell\ [e^{-5iK\ell} W'(-1) + e^{5\mu_+}\ W'_{f_+}(-1) + e^{5\mu_-}\ W'_{f_-}(-1)].$$

From above equations B_1^+ and B_1^- can be determined. It should be remembered that the response given by equation (3.15) is for the Ist bay only.

The response in the nth bay is given by

$$W_T^{(n)}(\xi) = e^{-i(n-1)K\ell}\ W(\xi) + e^{(n-1)\mu_+}\ W_{f_+}(\xi) + e^{(n-1)\mu_-} W_{f_-}(\xi) \qquad (3.17)$$

The response given by equation (3.19) is for the unit value of the loading parameter i.e. P = 1. This will be so if we consider $W_T^{(n)}(\xi)$ in the non-dimensional form (i.e. dividing by the span length ℓ). The second derivative of $W_T^n(\xi)$ gives the non-dimensional curvature (bending strain or stress). The amplitude of this quantity is nothing but the frequency response function $|H(f)|$ if we consider the bending strain as the output. In the present work, this $|H(f)|$ is calculated for various values of CV at the ,mid-point of the spans (i.e. at $\xi = 0$) both theoretically and experimentally.

CHAPTER - IV

RESULTS AND DISCUSSIONS

The test element used in the present work has the following physical and geometrical characteristics:

Outer diameter = 23 mm.

Inner diameter = 20 mm.

No. of span = 5

Span length = 304.8 mm.

Material - Mild Steel

1st natural frequency - 670 Hz

Modulus of elasticity, E = 210 GN/m^2

Figures (4.1, 4.2) show the auto correlation functions of pressure and vibration for different flow velocities. The curves decay with increasing delay time showing the randomness in the process. The curve become steep with increasing velocity. Since the slope of the auto correlation curve is an indication of frequency components present in a signal, it shows that higher frequency components are introduced at higher velocities. The amplitudes of the curves change their sign but the net area under the curve is positive satisfying the condition of positive definiteness.

Figure (4.3) shows the PSD curves of pressure signal. These results are computed in a digital computer using auto correlation data with the help of equation (1.2). It is seen that most of the energy is associated with low frequency components. The curves show peaks between 30 and 40 Hz which

are mainly due to eddies and turbulence. The peaks are shifting to the higher frequencies as the flow velocity increases. This shows the flow dependence of the pressure field. At higher flow velocity the eddy life time decreases there by showing shift in the frequency spectrum.

Figure (4.4) shows PSD curves of the vibration response. These results are also computed in the same manner as the pressure spectrum. These curves also show peaks between 30 and 40 Hz. This shows the turbulence and eddies are primarily responsible for exciting the rod.

Figure 4.5 shows cross correlation curves between two pressure transducers which are placed axially apart. The axial separation between the transducers is denoted by (d) while T_d represents the delay time corresponding to peak amplitude of the cross correlation curve. The convection velocity C_p is calculated from the simple relation, $C_p = \frac{d}{T_d}$. It is found that when the flow velocity is 49.6 cm/sec the convection velocity is 46 cm/sec. This shows that the disturbance in the pressure field is travelling with a velocity almost equal to flow velocity.

Figures (4.6, 4.7, 4.8, 4.9) show the frequency response function $H(\Omega)$ of the system. These results are computed from PSD data of pressure excitation and vibration response using equation (1.3). The curves are plotted against non-dimensional frequency $\Omega(= 2\pi f \ell^2 \sqrt{\frac{m}{EI}})$ for different non-dimensional

convection velocities CV $\left(= C_p^{\ell} \sqrt{\frac{m}{EI}}\right)$. Since the absolute measurement of the amplitude was not done, the experimental curves are compared with the theoretical curves after matching the peak amplitude value. It is seen that the relative magnitudes of peak and valley are almost same. On comparing the experimental results with those obtained from theory, it is found that there is a shift in peak frequency between the experimental curve and the theoretical one. The experimental curves show peaks at higher frequencies than the theoretical ones. This discrepancy may be due to the fact that the pressure has been measured on the wall of the outer pipe rather than on the surface of the test element. In such cases the measured pressure may not carry the true spectral information of the actual pressure. Moreover pressure field disturbance due to coupling between the acting pressure and the resulting vibration of the element has not been accounted in the theory. This effect seems to be quite important and produces a significant change in the pressure field across the flow.

However to clear this doubt a pressure field record on the surface of a 1" dia rod was taken and it has been found that the frequency response curve, using the above result matches better with the theoretical prediction (Figure 4.10). It should also be noted that in theory the supports of the test element are considered as simple line supports where as in practice some rotational stiffness is always present at the

supports. This discrepancy also tends to push the peak frequency to the higher value.

The peaks of the experimental curves shift to higher frequency with increasing convection velocity which is also predicted by the theory. The value of the loss loss factor (η), used to obtained the theoretical curves, has been computed by a trials and error approach. First an experimentally obtained curve of the frequency response function is matched with a theoretical curve for a particular value of damping. Taking this value into account other results have been predicted.

The frequency response curves show that there is not much difference between the response of the third span and the fifth span in the convection velocity range 0.03 to 0.05. This is also predicted by theory. But it is expected that there will be a mark difference between the response of the third bay and fifth if the convection velocity increases beyond a non-dimensional value 2.

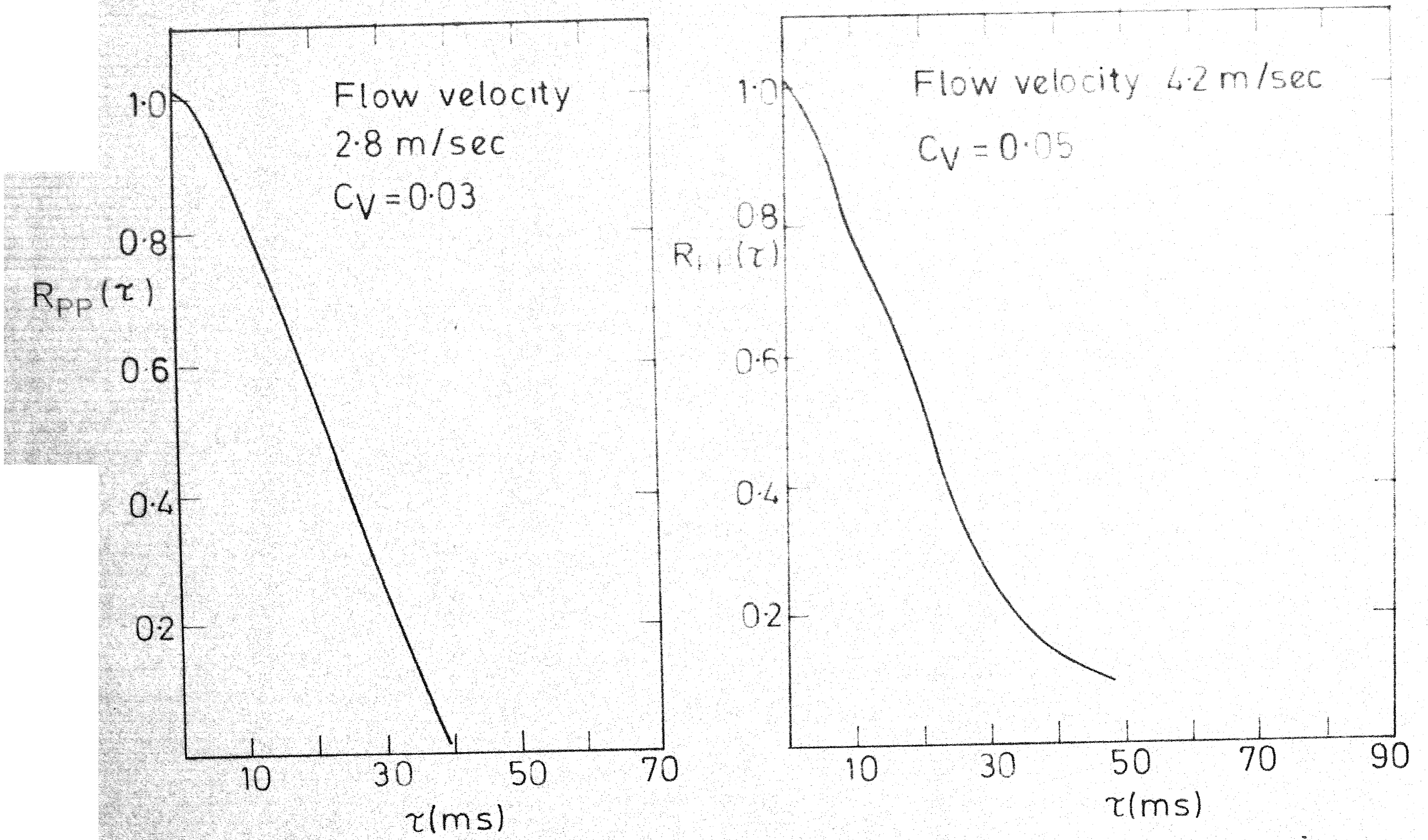


FIG. 4·1 PRESSURE AUTO CORRELATION FUNCTIONS AT THE WALL OF OUTER PIPE

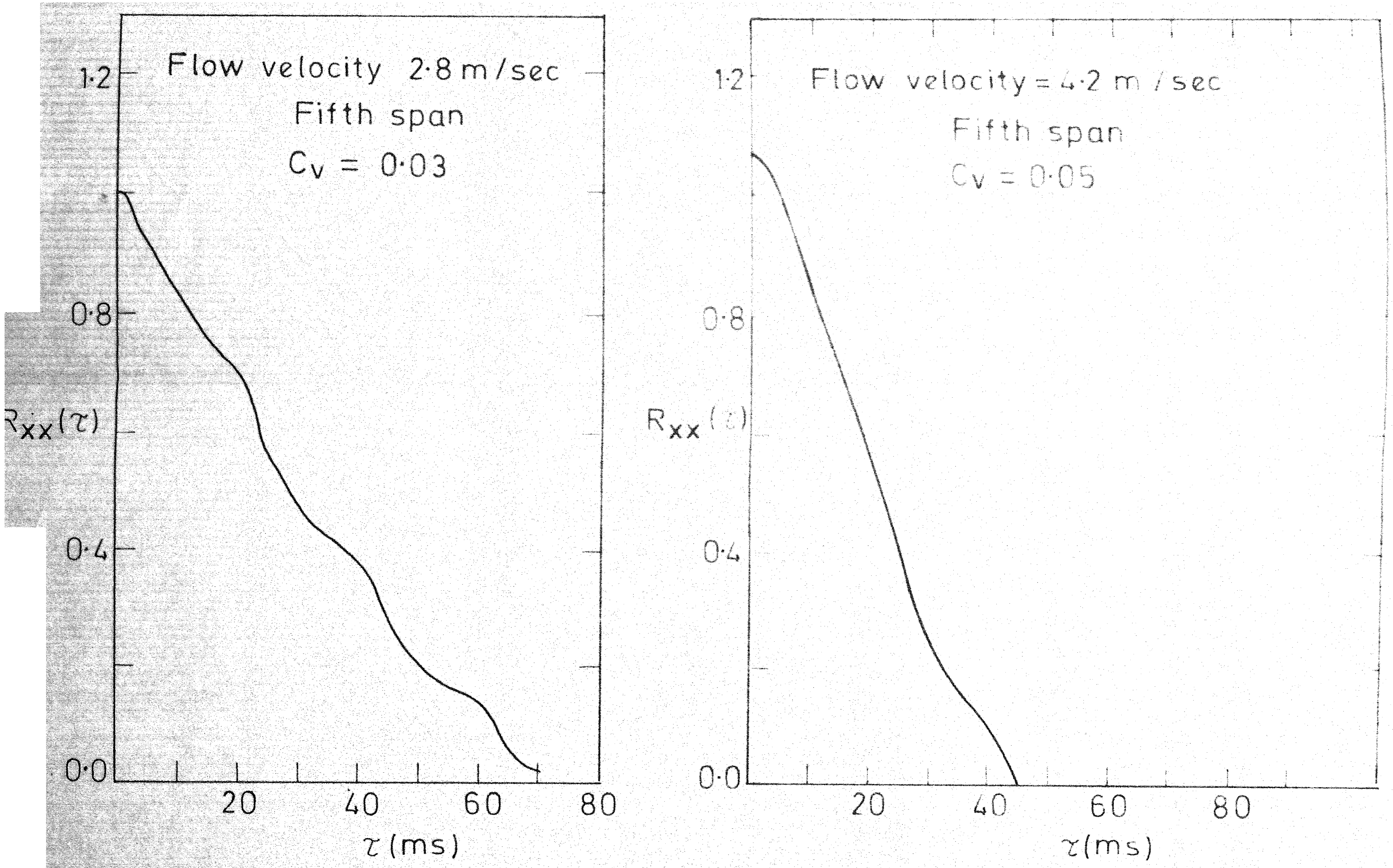


FIG. 4·2 VIBRATION AUTO CORRELATION FUNCTIONS OF THE TEST ELEMENT

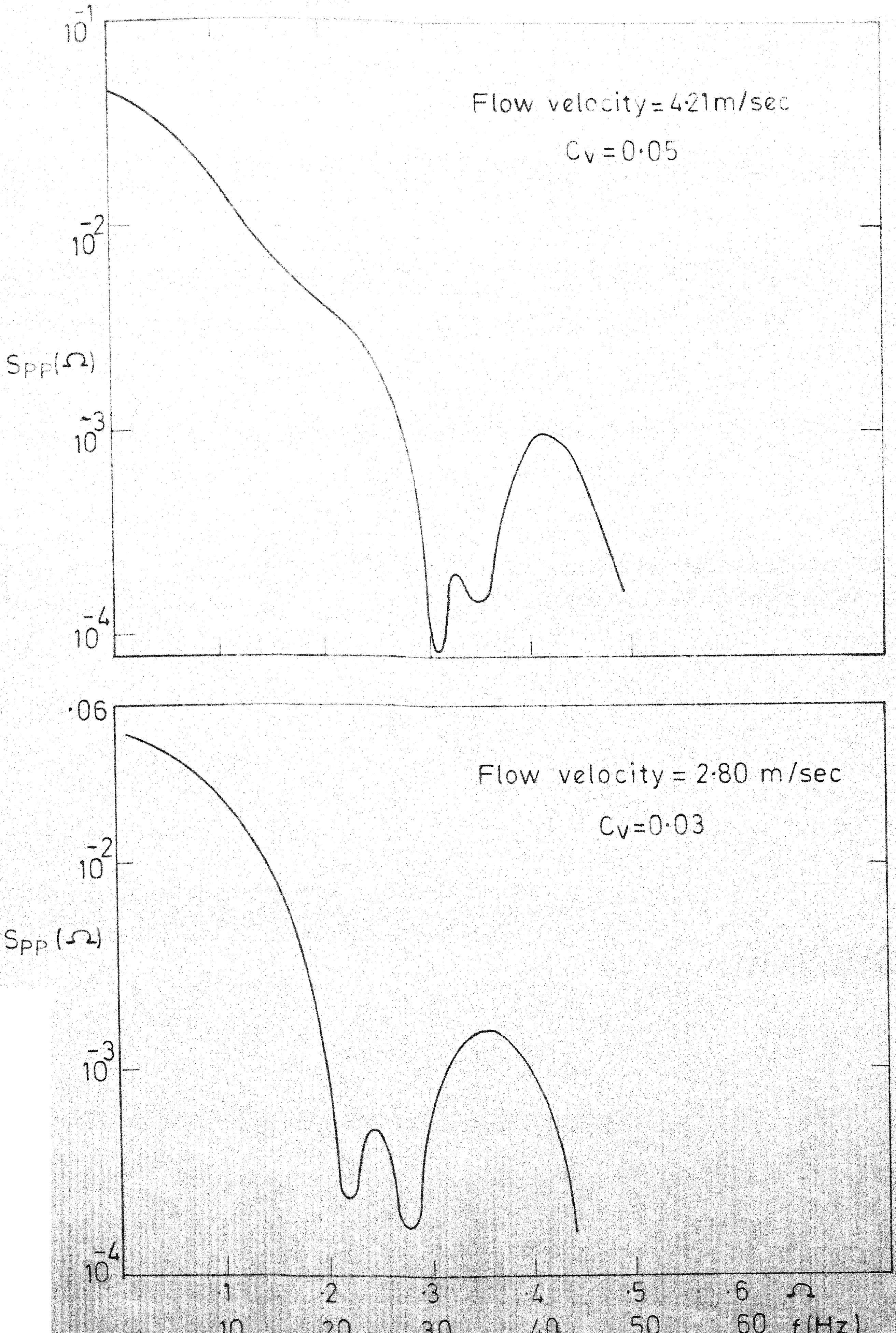


FIG. 4·3 PRESSURE SPECTRUM ON THE WALL OF THE OUTER PIPE

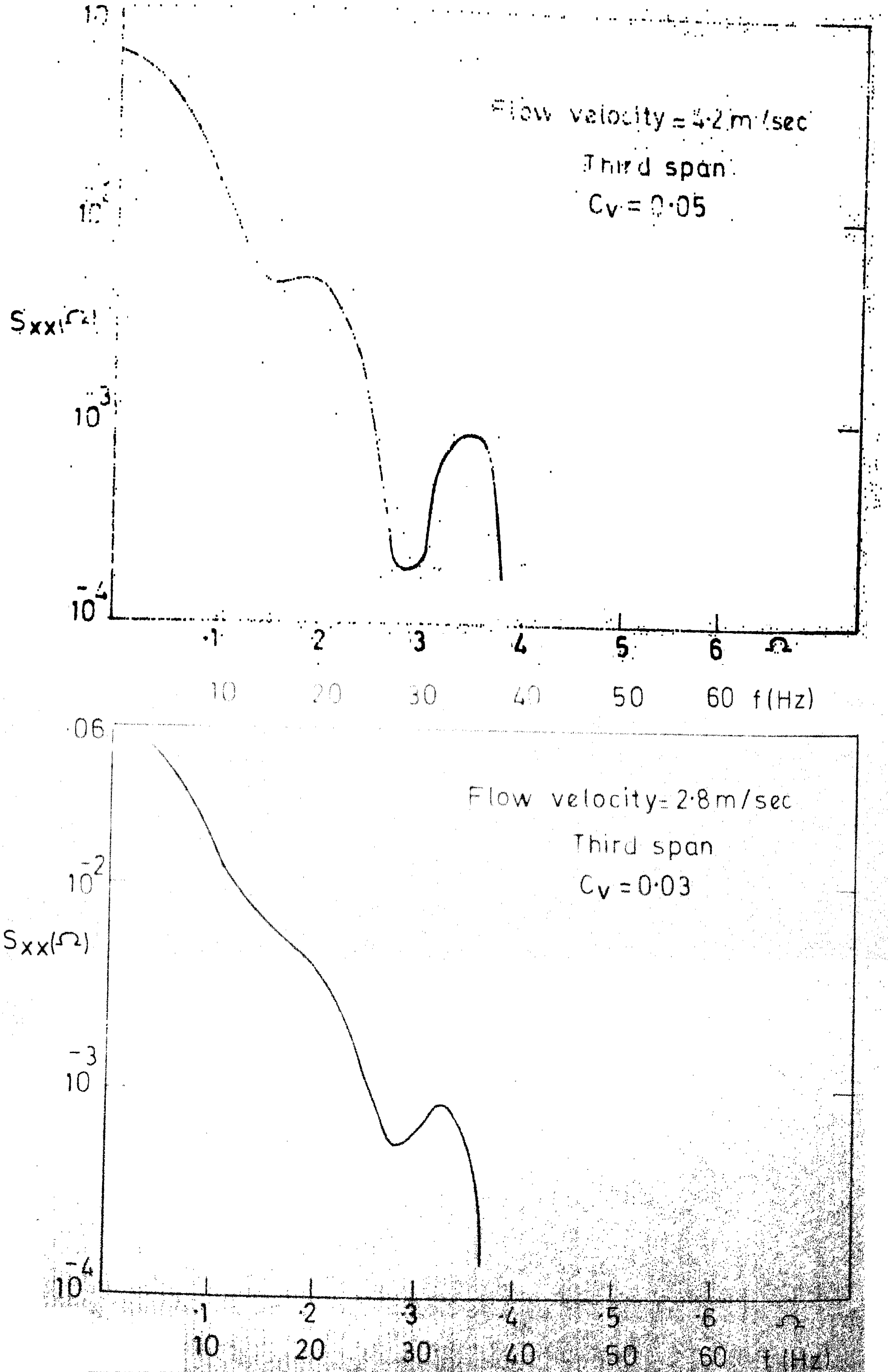


FIG 4.4 SPECTRUM OF THE VIBRATION OF TEST ELEMENT

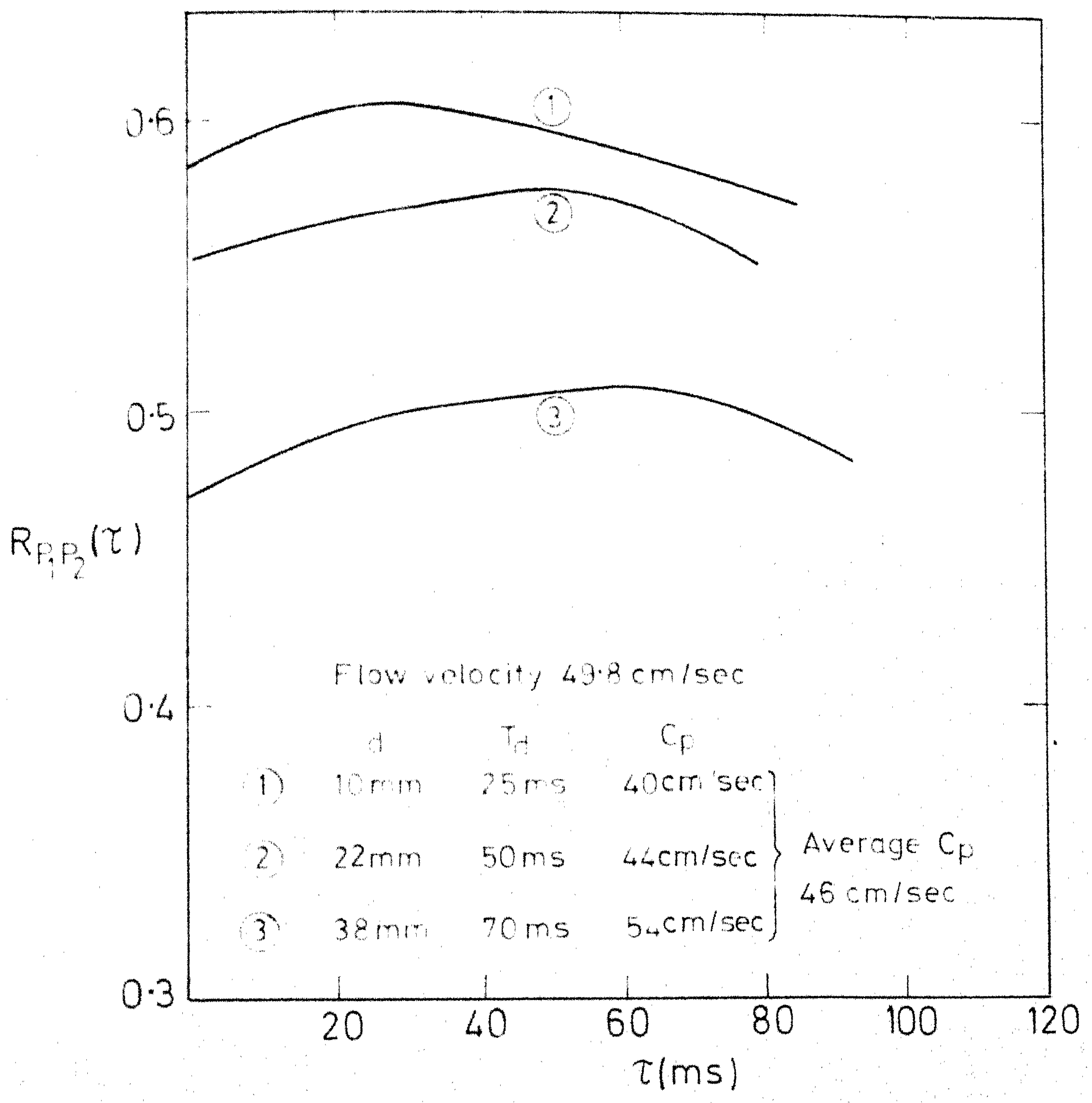


FIG. 4·5 CROSS CORRELATION CURVES OF TWO PRESSURE TRANSDUCERS PLACED AXIALLY APART

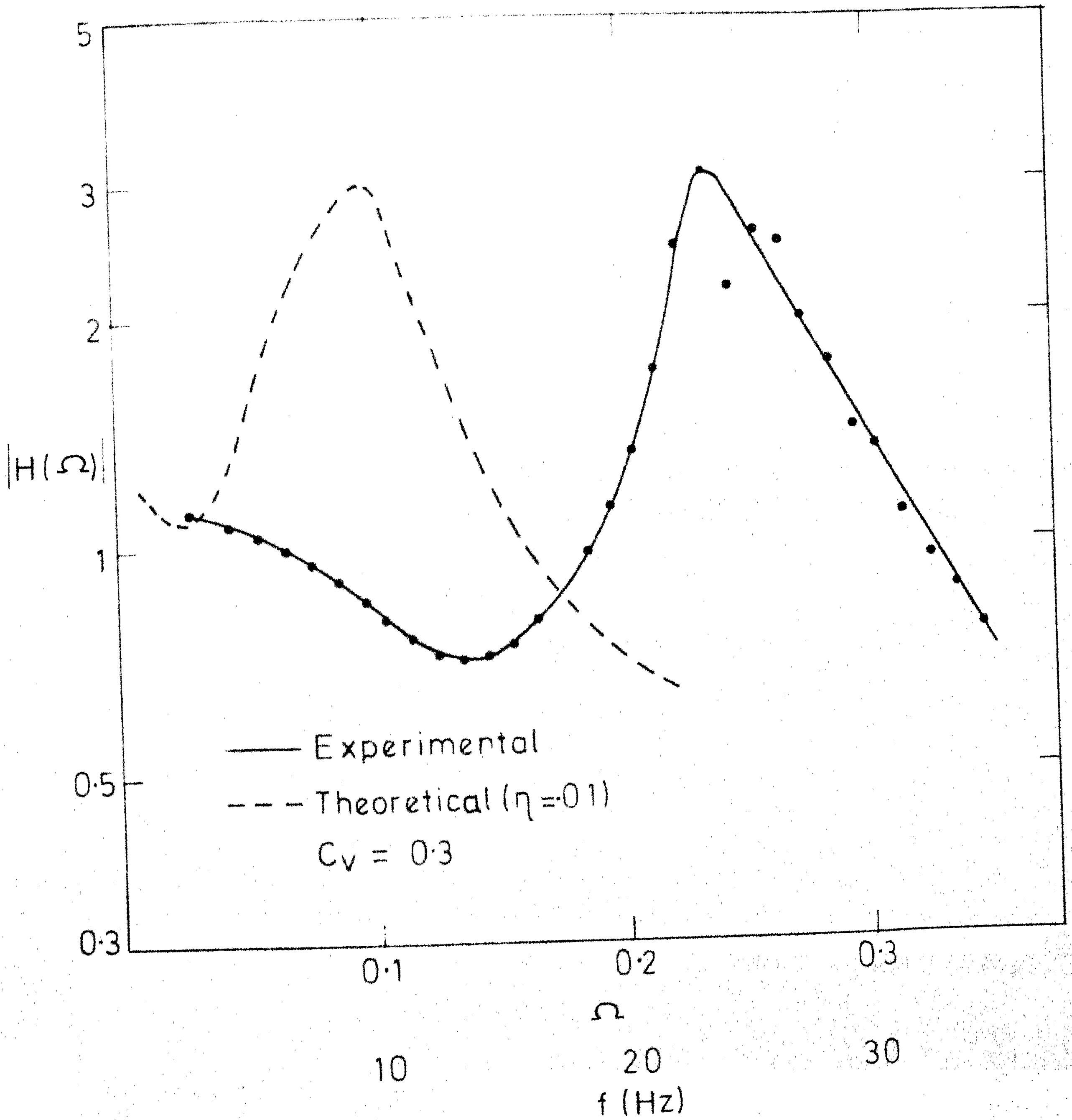


FIG. 4·6 FREQUENCY RESPONSE FUNCTION AT THE CENTRE OF THIRD BAY

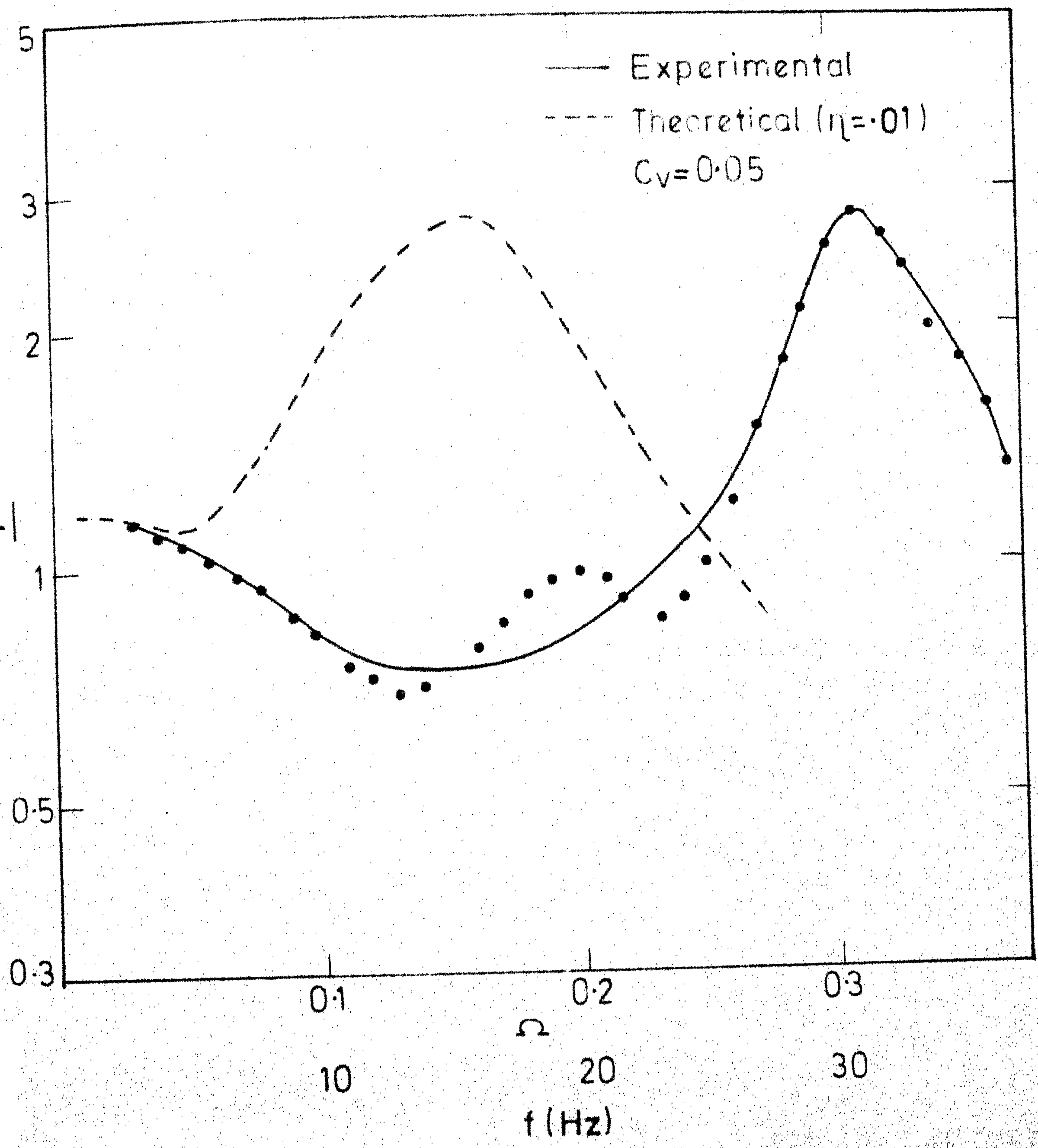


FIG. 4·7 FREQUENCY RESPONSE FUNCTION AT THE CENTRE OF THIRD BAY

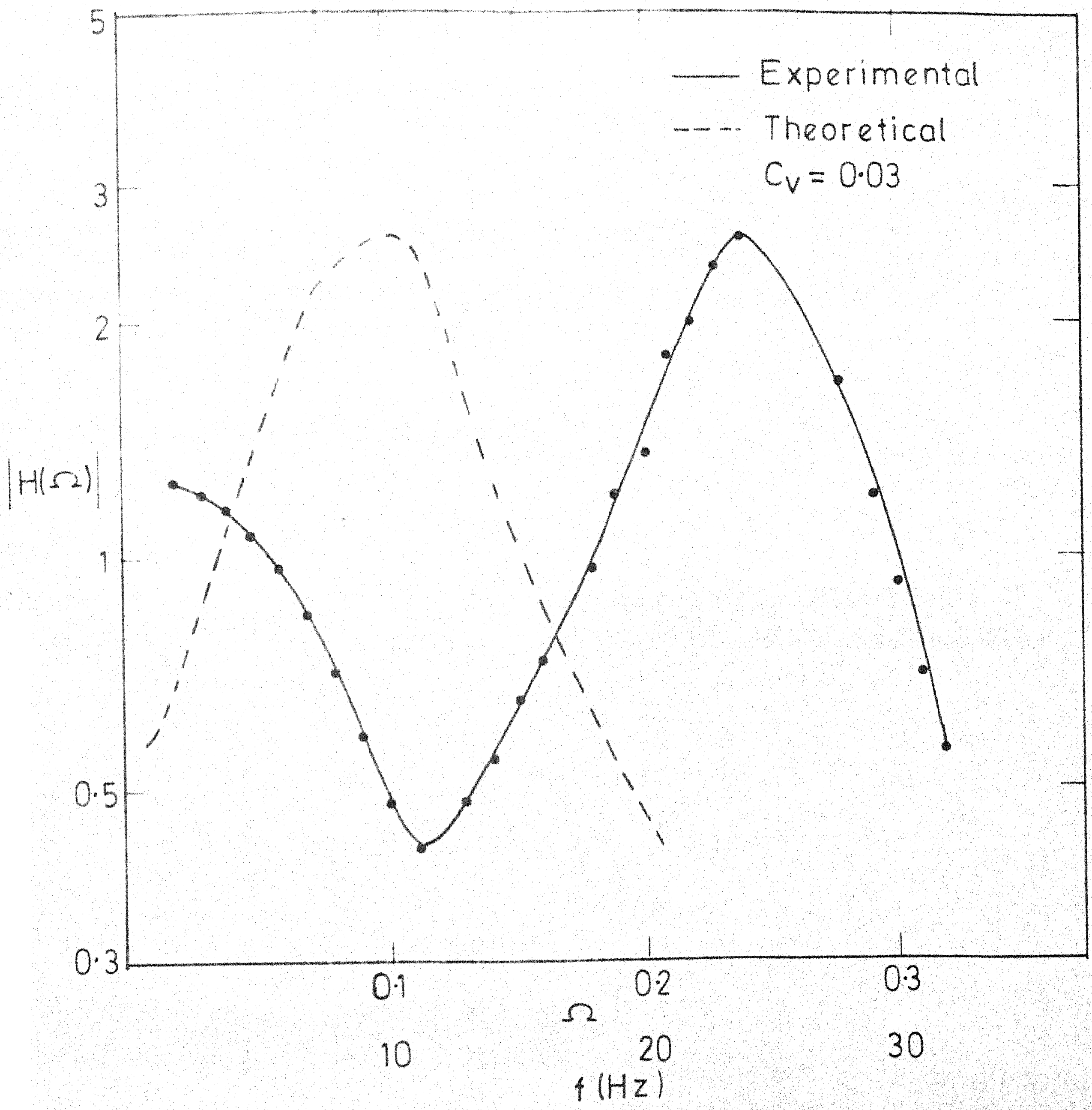


FIG. 4·8 FREQUENCY RESPONSE FUNCTION AT THE CENTRE OF FIFTH BAY

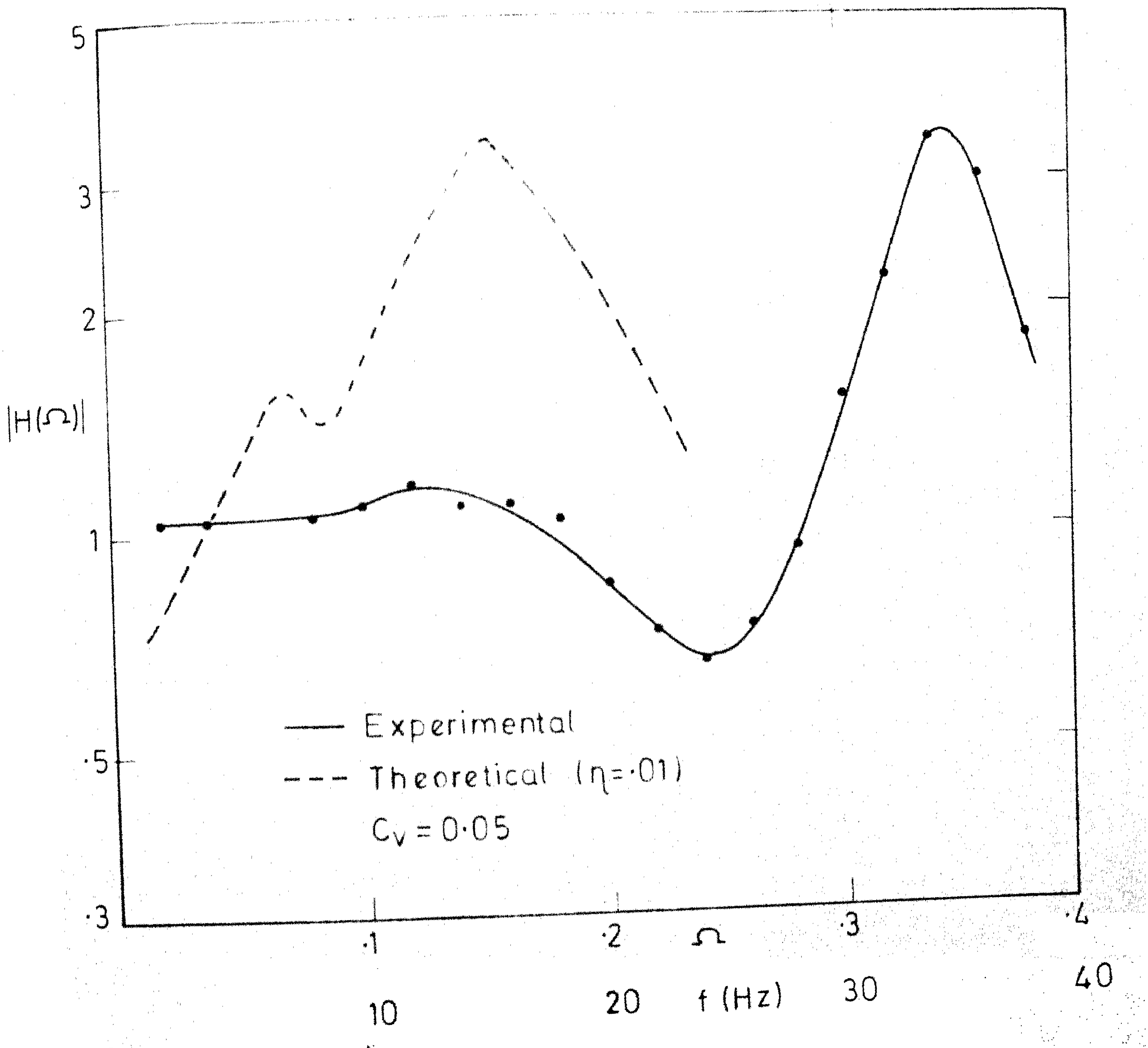
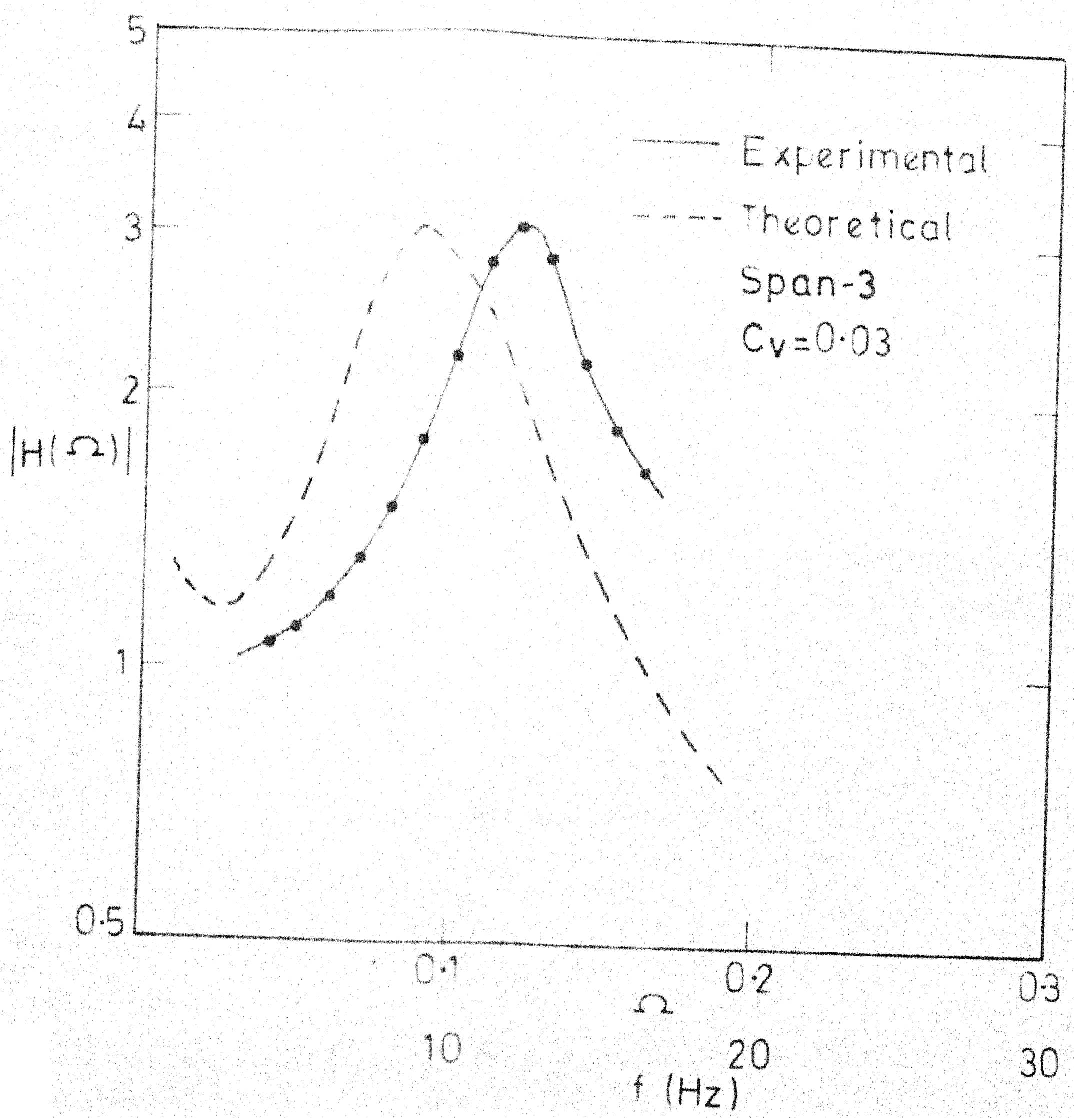


FIG. 4·9 FREQUENCY RESPONSE FUNCTION AT THE CENTRE OF FIFTH BAY



FIG· 4·10 FREQUENCY RESPONSE FUNCTION AT THE CENTRE OF THIRD BAY AFTER USING THE PRESSURE FIELD ON SURFACE OF TEST ELEMENT

CHAPTER - V

CONCLUSIONS

In the light of the above results the following conclusions can be drawn.

(i) Since the PSD and the frequency response curves are changing with the flow conditions, these can be considered as system representation rather than extraneous noise.

(ii) In the range of parameters of the present work, the vibration of the rod is forced type and highly flow dependent.

(iii) A periodically supported rod subjected to parallel flow may vibrate with high amplitude at a frequency much lower than its natural frequency. This peak frequency increases with increasing flow velocity.

(iv) The pressure field is mainly dependent on eddies which propagate with the flow velocity.

(v) Pressure field should be measured on the specimen even at this low flow velocity.

APPENDIX

Honeywell Correlator and Signal Analyser (Model SAI-48)

Product Description :

SAI-48 is a high speed processing instrument for on line correlation, signal averaging and probability computation. The outstanding features of this instruments are:

(1) 400 point analysis

(2) 5 MHz (0.2 μsec) sampling rate

(3) Digital exponential averaging

(4) 800 points precomp delay

(5) Digital selection and read out of any bin.

Operation :

(a) Correlation :

Correlation is a fundamental tool for time domain analysis SAI-48 provides auto and cross correlation function with incremental time lag ranging from 0.2 μsec to 2 sec. For both auto and cross the processing of signal is identical. The only difference occurs in input circuit. In cross correlation two different signals are applied to the two different channels of SAI-48. In auto correlation one input signal is applied to either channel and internally routed to the 2nd channel to be automatically processed as in Cross Correlation. Regarding of

whether the mode is auto or cross the operation is as follows:

The input signal is sampled at the rate indicated by chosen sample increment switch and quantized in both the channels. Channel I has got 400 word memory while channel II has got one word memory (the present sample). At time called present time, the correlator performs the multiplication of present word of channel II with each 400 word of channel I. The 400 results become the first entry of R(0), R(1)......R(399). R(399) stands for the correlation value at log time 399τ. The operation is repeated for repeatedly updated values in channel II and corresponding updated values in channel I and then averaged. For other details of operations like 'Enhancement, Probability density, Probability distribution' see Ref. [17].

Application :

In this experiment correlator has been used only for specific purpose like measurement of auto correlation and cross correlation of pressure and vibration signal of a rod in parallel flow. The cross correlation results gives the information about velocity while the auto correlation results are used to compute the PSD function. Honeywell has got a product 'SAI-470 Fourier Transformer' which can be connected to SAI-48 Correlator by interface connection to get the PSD function directly by Fourier analysis of correlation values.

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